Mathematical Models with Applications (IMRA25)

Subject: Mathematics
Grade: 10
Expectations: 37
Breakouts: 160

(a) Introduction.

1. The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century

2. The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication

3. Mathematical Models with Applications is designed to build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. This mathematics course provides a path for students to succeed in Algebra II and prepares them for various post-secondary choices. Students learn to apply mathematics through experiences in personal finance, science, engineering, fine arts, and social sciences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, model information, solve problems, and communicate solutions. Students will select from tools such as physical objects manipulatives technology, including graphing calculators, data collection devices, and computers paper and pencil and from methods such as algebraic techniques, geometric reasoning, patterns, and mental math to solve problems

4. In Mathematical Models with Applications, students will use a mathematical modeling cycle to analyze problems, understand problems better, and improve decisions. A basic mathematical modeling cycle is summarized in this paragraph. The student will

    a. represent:

        i. identify the variables in the problem and select those that represent essential features

        ii. formulate a model by creating and selecting from representations such as geometric, graphical, tabular, algebraic, or statistical that describe the relationships between the variables

    b. compute: analyze and perform operations on the relationships between the variables to draw conclusions

c. interpret: interpret the results of the mathematics in terms of the original problem

d. revise: confirm the conclusions by comparing the conclusions with the problem and revising as necessary

e. report: on the conclusions and the reasoning behind the conclusions

5. Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and Skills Statements

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(i) apply mathematics to problems arising in everyday life

(ii) apply mathematics to problems arising in society

(iii) apply mathematics to problems arising in the workplace

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(i) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem- solving process

(ii) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the reasonableness of the solution

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(i) select tools, including real objects as appropriate, to solve problems

(ii) select tools, including manipulatives as appropriate, to solve problems

(iii) select tools, including paper and pencil as appropriate, to solve problems

(iv) select tools, including technology as appropriate, to solve problems

(v) select techniques, including mental math as appropriate, to solve problems

(vi) select techniques including estimation as appropriate, to solve problems

(vii) select techniques, including number sense as appropriate, to solve problems

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(i) communicate mathematical ideas using multiple representations, including symbols as appropriate

(ii) communicate mathematical ideas using multiple representations, including diagrams as appropriate

(iii) communicate mathematical ideas using multiple representations, including graphs as appropriate

(iv) communicate mathematical ideas using multiple representations, including language as appropriate

(v) communicate mathematical reasoning using multiple representations, including symbols as appropriate

(vi) communicate mathematical reasoning using multiple representations, including diagrams as appropriate

(vii) communicate mathematical reasoning using multiple representations, including graphs as appropriate

(viii) communicate mathematical reasoning using multiple representations, including language as appropriate

(ix) communicate [mathematical ideas'] implications using multiple representations, including symbols as appropriate

(x) communicate [mathematical ideas'] implications using multiple representations, including diagrams as appropriate

(xi) communicate [mathematical ideas'] implications using multiple representations, including graphs as appropriate

(xii) communicate [mathematical ideas'] implications using multiple representations, including language as appropriate

(xiii) communicate [mathematical reasoning's] implications using multiple representations, including symbols as appropriate

(xiv) communicate [mathematical reasoning's] implications using multiple representations, including diagrams as appropriate

(xv) communicate [mathematical reasoning's] implications using multiple representations, including graphs as appropriate

(xvi) communicate [mathematical reasoning's] implications using multiple representations, including language as appropriate

(E) create and use representations to organize, record, and communicate mathematical ideas;

(i) create representations to organize mathematical ideas

(ii) create representations to record mathematical ideas

(iii) create representations to communicate mathematical ideas

(iv) use representations to organize mathematical ideas

(v) use representations to record mathematical ideas

(vi) use representations to communicate mathematical ideas

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(i) analyze mathematical relationships to connect mathematical ideas

(ii) analyze mathematical relationships to communicate mathematical ideas

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(i) display mathematical ideas using precise mathematical language in written or oral communication

(ii) display mathematical arguments using precise mathematical language in written or oral communication

(iii) explain mathematical ideas using precise mathematical language in written or oral communication

(iv) explain mathematical arguments using precise mathematical language in written or oral communication

(v) justify mathematical ideas using precise mathematical language in written or oral communication

(vi) justify mathematical arguments using precise mathematical language in written or oral communication

(2) Mathematical modeling in personal finance. The student uses mathematical processes with graphical and numerical techniques to study patterns and analyze data related to personal finance. The student is expected to:

(A) use rates and linear functions to solve problems involving personal finance and budgeting, including compensations and deductions;

(i) use rates to solve problems involving personal finance

(ii) use rates to solve problems involving budgeting, including compensations

(iii) use rates to solve problems involving budgeting, including deductions

(iv) use linear functions to solve problems involving personal finance

(v) use linear functions to solve problems involving budgeting, including compensations

(vi) use linear functions to solve problems involving budgeting, including deductions

(B) solve problems involving personal taxes; and

(i) solve problems involving personal taxes

(C) analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees.

(i) analyze data to make decisions about banking, including options for online banking

(ii) analyze data to make decisions about banking, including checking accounts

(iii) analyze data to make decisions about banking, including overdraft protection

(iv) analyze data to make decisions about banking, including processing fees

(v) analyze data to make decisions about banking, including debit card/ATM fees

(3) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. The student is expected to:

(A) use formulas to generate tables to display series of payments for loan amortizations resulting from financed purchases;

(i) use formulas to generate tables to display series of payments for loan amortizations resulting from financed purchases

(B) analyze personal credit options in retail purchasing and compare relative advantages and disadvantages of each option;

(i) analyze personal credit options in retail purchasing

(ii) compare relative advantages of each option

(iii) compare relative disadvantages of each option

(C) use technology to create amortization models to investigate home financing and compare buying a home to renting a home; and

(i) use technology to create amortization models to investigate home financing

(ii) use technology to create amortization models to compare buying a home to renting a home

(D) use technology to create amortization models to investigate automobile financing and compare buying a vehicle to leasing a vehicle.

(i) use technology to create amortization models to investigate automobile financing

(ii) use technology to create amortization models to compare buying a vehicle to leasing a vehicle

(4) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:

(A) analyze and compare coverage options and rates in insurance;

(i) analyze coverage options in insurance

(ii) analyze rates in insurance

(iii) compare coverage options in insurance

(iv) compare rates in insurance

(5) Mathematical modeling in science and engineering. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. The student is expected to:

(A) use proportionality and inverse variation to describe physical laws such as Hook's Law, Newton's Second Law of Motion, and Boyle's Law;

(i) use proportionality to describe physical laws

(ii) use inverse variation to describe physical laws

(B) use exponential models available through technology to model growth and decay in areas, including radioactive decay; and

(i) use exponential models available through technology to model growth in areas

(ii) use exponential models available through technology to model decay in areas, including radioactive decay

(C) use quadratic functions to model motion.

(i) use quadratic functions to model motion

(6) Mathematical modeling in science and engineering. The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. The student is expected to:

(A) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in architecture;

(i) use similarity to describe mathematical patterns in architecture

(ii) use similarity to describe mathematical structure in architecture

(iii) use geometric transformations to describe mathematical patterns in architecture

(iv) use geometric transformations to describe mathematical structure in architecture

(v) use symmetry to describe mathematical patterns in architecture

(vi) use symmetry to describe mathematical structure in architecture

(vii) use perspective drawings to describe mathematical patterns in architecture

(viii) use perspective drawings to describe mathematical structure in architecture

(B) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields;

(i) use scale factors with two-dimensional objects

(ii) use scale factors with three-dimensional objects to demonstrate proportional changes in surface area as applied to fields

(iii) use scale factors with three-dimensional objects to demonstrate proportional changes in volume as applied to fields

(iv) use scale factors with three-dimensional objects to demonstrate non-proportional changes in surface area as applied to fields

(v) use scale factors with three-dimensional objects to demonstrate non-proportional changes in volume as applied to fields

(C) use the Pythagorean Theorem and special right-triangle relationships to calculate distances; and

(i) use the Pythagorean Theorem to calculate distances

(ii) use special right-triangle relationships to calculate distances

(D) use trigonometric ratios to calculate distances and angle measures as applied to fields.

(i) use trigonometric ratios to calculate distances as applied to fields

(ii) use trigonometric ratios to calculate angle measures as applied to fields

(7) Mathematical modeling in fine arts. The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. The student is expected to:

(A) use trigonometric ratios and functions available through technology to model periodic behavior in art and music;

(i) use trigonometric ratios available through technology to model periodic behavior in art

(ii) use trigonometric ratios available through technology to model periodic behavior in music

(iii) use trigonometric functions available through technology to model periodic behavior in art

(iv) use trigonometric functions available through technology to model periodic behavior in music

(B) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography;

(i) use similarity to describe mathematical patterns in art

(ii) use similarity to describe mathematical patterns in photography

(iii) use similarity to describe mathematical structure in art

(iv) use similarity to describe mathematical structure in photography

(v) use geometric transformations to describe mathematical patterns in art

(vi) use geometric transformations to describe mathematical patterns in photography

(vii) use geometric transformations to describe mathematical structure in art

(viii) use geometric transformations to describe mathematical structure in photography

(ix) use symmetry to describe mathematical patterns in art

(x) use symmetry to describe mathematical patterns in photography

(xi) use symmetry to describe mathematical structure in art

(xii) use symmetry to describe mathematical structure in photography

(xiii) use perspective drawings to describe mathematical patterns in art

(xiv) use perspective drawings to describe mathematical patterns in photography

(xv) use perspective drawings to describe mathematical structure in art

(xvi) use perspective drawings to describe mathematical structure in photography

(C) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music; and

(i) use geometric transformations to describe mathematical patterns in music

(ii) use geometric transformations to describe mathematical structure in music

(iii) use proportions to describe mathematical patterns in music

(iv) use proportions to describe mathematical structure in music

(v) use periodic motion to describe mathematical patterns in music

(vi) use periodic motion to describe mathematical structure in music

(D) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields.

(i) use scale factors with two-dimensional objects

(ii) use scale factors with three-dimensional objects to demonstrate proportional changes in surface area as applied to fields

(iii) use scale factors with three-dimensional objects to demonstrate proportional changes in volume as applied to fields

(iv) use scale factors with three-dimensional objects to demonstrate non-proportional changes in surface area as applied to fields

(v) use scale factors with three-dimensional objects to demonstrate non-proportional changes in volume as applied to fields

(8) Mathematical modeling in social sciences. The student applies mathematical processes to determine the number of elements in a finite sample space and compute the probability of an event. The student is expected to:

(A) determine the number of ways an event may occur using combinations, permutations, and the Fundamental Counting Principle;

(i) determine the number of ways an event may occur using combinations

(ii) determine the number of ways an event may occur using permutations

(iii) determine the number of ways an event may occur using the Fundamental Counting Principle

(B) compare theoretical to empirical probability; and

(i) compare theoretical to empirical probability

(C) use experiments to determine the reasonableness of a theoretical model such as binomial or geometric.

(i) use experiments to determine the reasonableness of a theoretical model

(9) Mathematical modeling in social sciences. The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. The student is expected to:

(A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, dot plots, stem-and-leaf plots, and box and whisker plots, to draw conclusions from the data and determine the strengths and weaknesses of conclusions;

(i) interpret information from various graphs, including line graphs, to draw conclusions from the data

(ii) interpret information from various graphs, including bar graphs, to draw conclusions from the data

(iii) interpret information from various graphs, including circle graphs, to draw conclusions from the data

(iv) interpret information from various graphs, including histograms, to draw conclusions from the data

(v) interpret information from various graphs, including scatterplots, to draw conclusions from the data

(vi) interpret information from various graphs, including dot plots, to draw conclusions from the data

(vii) interpret information from various graphs, including stem- and-leaf plots, to draw conclusions from the data

(viii) interpret information from various graphs, including box and whisker plots, to draw conclusions from the data

(ix) determine the strengths of conclusions

(x) determine the weaknesses of conclusions

(B) analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions;

(i) analyze numerical data using measures of central tendency (mean, median, and mode) in order to make inferences with normal distributions

(ii) analyze numerical data using measures of variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions

(C) distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies;

(i) distinguish the purposes of research, including surveys

(ii) distinguish the purposes of research, including experiments

(iii) distinguish the purposes of research, including observational studies

(iv) distinguish the differences among types of research, including surveys

(v) distinguish the differences among types of research, including experiments

(vi) distinguish the differences among types of research, including observational studies

(D) use data from a sample to estimate population mean or population proportion;

(i) use data from a sample to estimate population mean or population proportion

(E) analyze marketing claims based on graphs and statistics from electronic and print media and justify the validity of stated or implied conclusions; and

(i) analyze marketing claims based on graphs from electronic media

(ii) analyze marketing claims based on graphs from print media

(iii) analyze marketing claims based on statistics from electronic media

(iv) analyze marketing claims based on statistics from print media

(v) justify the validity of stated or implied conclusions

(F) use regression methods available through technology to model linear and exponential functions, interpret correlations, and make predictions.

(i) use regression methods available through technology to model linear functions

(ii) use regression methods available through technology to model exponential functions

(iii) use regression methods available through technology to interpret correlations

(iv) use regression methods available through technology to make predictions

(10) Mathematical modeling in social sciences. The student applies mathematical processes to design a study and use graphical, numerical, and analytical techniques to communicate the results of the study. The student is expected to:

(A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions; and

(i) formulate a meaningful question

(ii) determine the data needed to answer the question

(iii) gather the appropriate data

(iv) analyze the data

(v) draw reasonable conclusions

(B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation.

(i) communicate methods used for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation

(ii) communicate analyses conducted for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation

(iii) communicate conclusions drawn for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation